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BINARY LAMINAR BOUNDARY LAYER OVER A VERTICAL SURFACE WITH COMBINED FREE AND FORCED CONVECTION

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BINARY LAMINAR BOUNDARY LAYER OVER A VERTICAL SURFACE WITH COMBINED FREE AND FORCED CONVECTION

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ABSTRACT. The study calculates the location of three types of motion modes, free, combined and forced convection, while examining the dynamic and thermal characteristics of a boundary layer with respect to them. Mass transfer with free and forced convections are then indicated.

We consider heat and mass transfer over a vertical surface with combined free and forced convection. The boundary layer differential equations, transformed to ordinary differential equations, contain a parameter which defines the influence of free convection on forced motion. Criteria are given for classifying the nature of the motion into purely free, purely forced convection and combined mode of motion. /120*

Notation

x, y - coordinates,
u, v - velocity components
g - gravitational acceleration,
T - temperature,
ν - kinematic viscosity,
β - thermal expansion coefficient,
a - thermal diffusivity coefficient,
ρ_1 - partial vapor density,
D - diffusion coefficient,
W_2 - weight velocity
η - independent variable
τ_w - shear stress at wall,
λ - thermal conductivity coefficient,
r - latent heat of phase transform,
θ, φ - dimensionless temperature and dimensionless partial vapor density,
m^* - complex $(m_{1\infty} - m_{1w})/(1 - m_{1w})$,
c_p - specific heat at constant pressure,
G - Grashof number,
R - Reynolds number,
P - Prandtl number,
S - Schmidt number,

*Numbers in the margin indicate pagination in the foreign text.

The index w denotes the value on the surface, the index ∞ denotes the value at a large distance from the surface, the index 1 denotes vapor, the index 2 denotes air. /121

We consider (Figure 1) a vertical plate with a constant temperature T_w and a partial density ρ_{1w} of component 1, located in the stream of a binary mixture flowing with velocity U_∞ in the direction of the lifting forces.

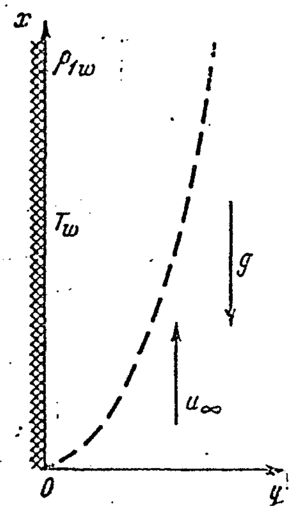

Figure 1

For a laminar incompressible boundary layer for which we neglect the viscous dissipation and for which we do not take into account the thermal diffusion, i. e., the diffusive thermal conductivity, and also under the assumptions that $c_{p1} = c_{p2}$ and that the physical parameters for the vertical plane are constant, the differential equations are written in the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta (T - T_\infty) \tag{1}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial y^2}, \quad u \frac{\partial \rho_1}{\partial x} + v \frac{\partial \rho_1}{\partial y} = D \frac{\partial^2 \rho_1}{\partial y^2}$$

with the following boundary conditions:

$$\begin{gathered} u = 0, \quad v = v_w, \quad T = T_w, \; \rho_1 = \rho_{1w} \text{ when } y = 0 \\ u = U_\infty, \; T = T_\infty, \; \rho_1 = \rho_{1\infty} \text{ when } y = \infty \end{gathered} \tag{2}$$

Let us also admit that the velocity of the condensed fluid on the wall and its thermal resistance are negligibly small in comparison with the velocity of unperturbed flow and the thermal resistance of the boundary layer. Furthermore, in the equations of motion we omit the term for the lift arising from the difference in concentrations.

We reduce system (1) to ordinary differential equations by introducing the independent variable η and the stream function ψ

$$\eta = y \left(\frac{U_\infty}{\nu x} \right)^{1/2}, \quad \Psi = (U_\infty \nu x)^{1/2} f(\eta) \quad \left(u = \frac{\partial \Psi}{\partial y}, \quad v = -\frac{\partial \Psi}{\partial x} \right) \tag{3}$$

In the new variables we have

$$u = U_\infty f'(\eta), \quad v = -\frac{1}{2}\left(\frac{U_\infty \nu}{x}\right)^{1/2} [f(\eta) - \eta f'(\eta)] \tag{4}$$

Then in the place of system (1) we obtain

$$f'''(\eta) + \frac{1}{2} f(\eta) f''(\eta) + \frac{G}{R^2}\theta(\eta) = 0$$
$$\theta''(\eta) + {}^1/_2 P f(\eta)\theta'(\eta) = 0, \quad \varphi''(\eta) + {}^1/_2 S f(\eta)\varphi'(\eta) = 0 \tag{5}$$

where

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \varphi(\eta) = \frac{\rho_1 - \rho_{1\infty}}{\rho_{1w} - \rho_{1\infty}}.$$

In the transformed equations there appears the parameter $G/R^2 = A$ which is independent of η. When this parameter is equated to zero equation (5) turns into the equation for forced convection; for large values of A, obviously, the free convection process will be dominant. The primes denote differentiation with respect to η.

In the new variables the boundary conditions (2) for system (5) will be:

$$\begin{gathered} f'(0) = 0, \quad f_w = \text{const}, \quad \theta = 1, \quad \varphi = 1 \text{ when } \eta = 0 \\ f'(\infty) = 1, \quad \theta = 0, \quad \varphi = 0 \text{ when } \eta = \infty \end{gathered} \tag{6}$$

The boundary condition f_w = const signifies that

$$v_w = -\frac{1}{2} f_w \left(\frac{U_\infty \nu}{x}\right)^{1/2}, \text{ i.e. } v_w \sim x^{-1/2}$$

The stated constraint does not manifest itself on the generality of the conclusions made in this paper. As was shown in [1, 2] the law of variation of v_w under free and forced convection has a comparatively weak influence on the variation of the characteristics of the boundary layer and of the local heat exchange coefficient. In the majority of cases the law $v_w \sim x^{-1/2}$ is consistent with the condition of constant temperature and constant mass content on the surface. The weight velocities of air and vapor on the plate's surface are /122

$$\begin{aligned} W_{2w} &= -D\left(\frac{\partial \rho_2}{\partial y}\right)_{y=0} - v_w \rho_{2w} = 0 \\ W_{1w} &= -D\left(\frac{\partial \rho_1}{\partial y}\right)_{y=0} - v_w \rho_{1w} \end{aligned} \tag{7}$$

which gives

$$v_w = -\frac{D}{\rho_{2w}}\left(\frac{\partial \rho_2}{\partial y}\right)_{y=0} = -\frac{D}{1-m_{1w}}\left(\frac{\partial m_1}{\partial y}\right)_{y=0} \tag{8}$$

since

$$m_1 + m_2 = 1, \quad m_1 = \rho_1/\rho, \quad m_2 = \rho_2/\rho$$

and, consequently,

$$W_{1w} = -\rho D \frac{1}{1-m_{1w}}\left(\frac{\partial m_1}{\partial y}\right)_{y=0} \tag{9}$$

From relations (3), (6), (8) and (9) we find that

$$f_w = -\frac{2}{S}\frac{m_{1\infty} - m_{1w}}{1-m_{1w}}\varphi'(0) \tag{10}$$

where m_{1w} is the mass content of the vapor on the surface.

The nonlinear system (5) with boundary conditions (6) was solved on the electronic computer M-20 by an iteration method using the sweep technique [3, 4]. The Blasius solution [5] was taken as the zeroth approximation for $f(\eta)$.

As a result of the computations carried out we obtained the profiles for the velocity, the temperature and the distribution of the partial density of component 1 in the boundary layer (Figures 2, 3, 4) for mixed free and forced convection for the numbers P = 0.72, S = 0.6 and for the parameter A equal to 0.1, 1, 10, 100 with $f_w = 0.05$. On Figure 4 as an example is shown the influence of the mass flow in the temperature profile for A = 0.1. The solid lines on Figure 3 represent the distribution of the density of component 1, while the dashed lines represent the temperature distribution.

A. A. Szewczyk [6] solved the problem for the case $f_w = 0$, but without taking the phase transforms into account. In this case the results in the present paper coincide with those in [6].

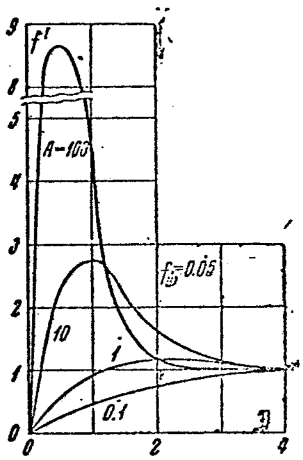

Figure 2

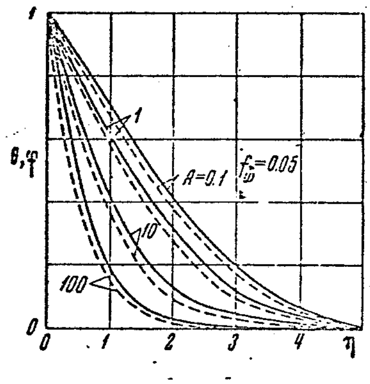

Figure 3

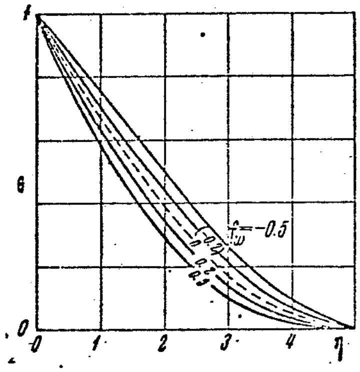

Figure 4

The local skin friction on the wall is determined by the expression

$$\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} - \rho v_w U_\infty \tag{11}$$

From relation (4) we get that

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = U_\infty \left(\frac{U_\infty}{\nu x}\right)^{1/2} f''(0).$$

Then from (11) we have /123

$$\tau_w = \mu \frac{U_\infty^2}{\nu^2} \frac{f''(0)}{R^{1/2}} + \frac{1}{2} \rho U_\infty^2 \frac{f_w(0)}{R^{1/2}}$$

or, in dimensionless form,

$$\frac{\tau_w}{\rho U_\infty^2} = \frac{f''(0)}{R^{1/2}} + \frac{1}{2} \frac{f_w(0)}{R^{1/2}} \tag{12}$$

The total heat flux (with due regard to the phase transition heat), withdrawn through the wall is computed from the formula

$$q_w^* = -\lambda \left(\frac{\partial T}{\partial y}\right)_{y=0} \pm \frac{r\rho D}{1 - m_{1w}} \left(\frac{\partial m_1}{\partial y}\right)_{y=0} \tag{13}$$

and, moreover, we take the plus sign for evaporation and the minus sign for condensation.

Then the heat removal coefficient (with due regard to the phase transition heat) is

$$\alpha^* = \frac{q_w^*}{T_w - T_\infty} = -\lambda R^{1/2} x^{-1} \left\{ \theta'(0) \pm \frac{P}{S} \frac{r m^*}{c_p (T_w - T_\infty)} \varphi'(0) \right\} \tag{14}$$

or

$$N = \frac{\alpha^* x}{\lambda} = -R^{1/2} \left\{ \theta'(0) \pm \frac{P}{S} \frac{r m^*}{c_p (T_w - T_\infty)} \varphi'(0) \right\} \tag{15}$$

The first term occurring in the curly brackets gives the convective component of the heat flux. Analogously, for the mass flux

$$j_1 = -\rho D \left(\frac{\partial m_1}{\partial y} \right)_{y=0} = -\rho D \left(\frac{U_\infty}{\nu x} \right)^{1/2} (m_{1w} - m_{1\infty}) \varphi'(0)$$

or the mass removal coefficient is

$$\alpha_m = \frac{j_1}{\rho (m_{1w} - m_{1\infty})} = -D R^{1/2} x^{-1} \varphi'(0) \tag{16}$$

and, consequently, the Nusselt number for mass transfer will be

$$N_D = \frac{\alpha_m x}{D} = -R^{1/2} \varphi'(0) \tag{17}$$

The conditions under which the heat removal process can be treated either as only a free convection flow or as only a forced convection flow may be determined by comparing the numerical calculation with the results of the heat removal calculation for purely forced and purely free convections with $P = 0.72$ by means of the equations

$$\frac{N}{R^{1/2}} = 0.297, \quad \frac{N}{R^{1/2}} = -\frac{\theta'(0)}{\sqrt{2}} \left(\frac{G}{R^2} \right)^{1/4} \tag{18}$$

The first relation in (18) was obtained from the results of this paper and coincides with that given by other authors [1], while the second relation in (18) was obtained from the results of [7].

If we take it [8] that the heat transfer under purely forced or purely free convection differs from (18) by no more than 5%, then the boundaries of these streams can be determined from the conditions

$0 < A < 0.095$	-	forced convection
$0.095 < A < 16$	-	combined convection
$16 < A$	-	free convection

Analogously, from the well-known expressions [5, 7]

$$\tfrac{1}{2} C_f R^{1/2} = 0.323, \quad \tfrac{1}{2} C_f R^{1/2} = \sqrt{2}\, f''(0) \left(\frac{G}{R^2}\right)^{3/4} \qquad (19)$$

we can obtain the boundaries of the stream modes for calculating the skin friction

$0 < A < 0.015$ — forced convection
$0.015 < A < 16$ — combined convection
$16 < A$ — free convection

FREE AND FORCED CONVECTIONS COINCIDING IN DIRECTION

f_w	$\theta'(0)$ $P = 0.72$	$S = 0.6$ $\varphi'(0)$	$S = 0.6$ m^*	$S = 0.9$ $\varphi'(0)$	$S = 0.9$ m^*
		$A = 0.1$			
0.06	—0.3295	—0.3105	0.0579	—0.3561	0.0758
0.04	—0.3246	—0.3064	0.0391	—0.3499	0.0514
0.02	—0.3198	—0.3024	0.0198	—0.3439	0.0263
0.0	—0.3149	—0.2984	0.0	—0.3380	0.0
—0.02	—0.3102	—0.2944	—0.0203	—0.3319	—0.0271
—0.04	—0.3454	—0.2904	—0.0413	—0.3259	—0.0552
—0.06	—0.3006	—0.2864	—0.0628	—0.3201	—0.0343
		$A = 1$			
0.06	—0.4125	—0.3835	0.0469	—0.4518	0.0597
0.04	—0.4082	—0.3799	0.0316	—0.4464	0.0403
0.02	—0.4039	—0.3764	0.0159	—0.4409	0.0204
0.0	—0.3998	—0.3730	0.0	—0.4359	0.0
—0.02	—0.3956	—0.3696	—0.0162	—0.4306	—0.0209
—0.04	—0.3913	—0.3660	—0.0361	—0.4252	—0.0423
—0.06	—0.3871	—0.3625	—0.0480	—0.4199	—0.0643
		$A = 10$			
0.06	—0.6393	—0.5881	0.0306	—0.7064	0.0382
0.04	—0.6354	—0.5849	0.0205	—0.7014	0.0256
0.02	—0.6315	—0.5315	0.0103	—0.6964	0.0129
0.0	—0.6277	—0.5786	0.0	—0.6916	0.0
—0.02	—0.6235	—0.5754	—0.0104	—0.6866	—0.0132
—0.04	—0.6201	—0.5722	—0.0209	—0.6817	—0,0264
—0.06	—0.6162	—0.5631	—0.0316	—0.6768	—0.0399
		$A = 100$			
0.06	—1.0756	—0.9376	0.0182	—1.1889	0.0227
0.04	—1.0716	—0.9344	0.0122	—1.1837	0.0152
0.02	—1.0679	—0.9313	0.0061	—1.1788	0·0076
0.0	—1.0641	—0.9783	0.0	—1.1741	0.0
—0.02	—1.0603	—0.9751	—0.0061	—1.1692	—0.0077
—0.04	—1.0564	—0.9720	—0.0123	—1.1643	—0.0154
—0.06	—1.0526	—0.9689	—0.0186	—1.1594	—0.0233

FREE AND FORCED CONVECTIONS OPPOSING IN DIRECTION

			$A = 0.25$		
0.06	—0.2162	—0.2138	0.0341	—0.2199	0.1224
0.04	—0.2133	—0.2114	0.0567	—0.2162	0.0933
0.02	—0.2104	—0.2030	0.0237	—0.2124	0.0424
0.0	—0.2076	—0.2067	0.0	—0.2090	0.0
—0.02	—0.2047	—0.2043	—0.0294	—0.2053	—0.0434
—0.04	—0.2019	—0.2019	—0.0594	—0.2018	—0.0891
—0.06	—0.1991	—0.1996	—0.0902	—0.1982	—0.1361
			$A = 1$		
0.06	—0.3037	—0.2904	0.0620	—0.3359	0.0804
0.04	—0.3054	—0.2878	0.0417	—0.3317	0.0543
0.02	—0.3021	—0.2852	0.0211	—0.3275	0.0275
0	—0.2939	—0.2825	0.0	—0.3235	0.0
—0.02	—0.2959	—0.2801	—0.0214	—0.3193	—0.0282
—0.04	—0.2927	—0.2775	—0.0433	—0.3152	—0.0571
—0.06	—0.2895	—0.2749	—0.0655	—0.3111	—0.0868
			$A = 10$		
0.06	—0.5849	—0.5311	0.0339	—0.6562	0.0412
0.04	—0.5814	—0.5282	0.0227	—0.6515	0.0276
0.02	—0.5778	—0.5253	0.0114	—0.6468	0.0139
0.0	—0.5742	—0.5224	0.0	—0.6421	0.0
—0.02	—0.5706	—0.5195	—0.0115	—0.6375	—0.0141
—0.04	—0.5671	—0.5167	—0.0232	—0.6328	—0.0284
—0.06	—0.5635	—0.5138	—0.0350	—0.6282	—0.0429
			$A = 100$		
0.06	—1.0470	—0.9552	0.0188	—1.1636	0.0232
0.04	—1.0432	—0.9528	0.0126	—1.1587	0.0155
0.02	—1.0395	—0.9492	0.0063	—1.1539	0.0078
0	—1.0358	—0.9462	0.0	—1.1492	0.0
—0.02	—1.0332	—0.9432	—0.0064	—1.1445	—0.0079
—0.04	—1.0284	—0.9402	—0.0128	—1.1397	—0.0158
—0.06	—1.0247	—0.9372	—0.0192	—1.1348	—0.0240

The conclusions made above concerning the locations of the three types of motion modes are preserved also for the case when $f_w \neq 0$.

After the study of the dynamical and thermal characteristics of the boundary layer we pass on to consider the problem of mass transfer under the combined action of free and forced convections. In the Table we have presented $\theta'(0)$, $\phi'(0)$ and m^* for various values of the parameter A with the values P = 0.72, S = 0.9 and 0.6 both for condensation as well as for evaporation. For $f_w > 0$ we have condensation, while when $f_w < 0$, evaporation from the vertical surface.

Calculations have been carried out also for the case when the forced motion does not coincide with the direction of the free convection motion but is opposite in direction. In this case we have $f'(\infty) = -1$, in boundary conditions (6) when $\eta = \infty$, while the remaining conditions are unchanged. The system of equations (5) also remain unaltered.

Expressions (12), (15) and (17) for the skin friction and for the local Nusselt numbers remain valid for the process characterized by free and forced convections which are opposite in direction.

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REFERENCES

1. Eckert, E. R., and R. M. Drake Jr., "Heat and Mass Transfer," 2nd Edition, McGraw-Hill Book Co. Inc., New York, 1959. (Russian translation: Goxenergoizdat, Moscow-Leningrad, 1961).

2. Brdlik, R. M. and V. A. Mochalov, "Experimental Study of Free Convection with Porous Blowing and Suction on a Vertical Surface," *Inzh.-Fiz. Zh.*, Vol. 10, No. 1, 1966.

3. Berezin, I. S. and N. P. Zhidkov, *Metody vychislyenij* [Computational Methods], Fizmatgiz, Moscow, Vol. 2, 1959.

4. Brdlik, P. M., V. A. Mochalov and V. I. Dubovik, "Laminary Free Convection on a Vertical Surface, Complicated by Condensation or Evaporation," *Nauchn. Tr. Nauchno-Issled. Inst. Stroit. Fiz. Gosstroya SSSR*, No. 2, 1967.

5. Schlichting, H., "Boundary Layer Theory," 4th Edition, McGraw-Hill Book Co. Inc., New York, 1960. (Russian translation: *Izd. Inostr. Lit.*, Moscow, 1956.

6. Szewczyk, A. A., "Combined Forced and Free Convection Laminar Flow," *Trans. ASME, Ser. C. J. Heat Transfer*, No. 4, 1964. (Russian translation: *Teploperedacha, Tr. Amerik. Obshch. Inzh. Mekhan.*, Ser. C, No. 4, 1964.

7. Octrach, S., "An Analysis of Laminar Free Convection Flow and Heat Transfer About a Flat Plate Parallel to the Direction of the Generating Body Force," *NACA*, TR 1111, 1953.

8. Sparrow, E. M., R. Eichorn and J. L. Gregg, "Combined Forced and Free Convection in a Boundary Layer Flow," *Phys. Fluids*, Vol. 2, No. 3, 1959.

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